

# Symbolic Simulation By Nodal Method Of Effective Branch Current Source

I. I. Okonkwo, P. I. Obi, G. C Chidolue, S. S. N. Okeke

**Abstract—** State variable are crucial to transient simulation initialization of the electrical circuits. Other variable that are mathematically related to state variable may also be used to initialize transient response especially in symbolic simulation. In this paper we formulated and simulated electrical circuit transient symbolically using the transient nodal equation derived from branch effective current source. The result of the simulation agreed with the existing Matlab simpowersystem tool.

**Index Terms—** transient simulation, state variable, symbolic simulation, transient nodal equation, transient response, and electrical circuits.

## 1 INTRODUCTION

Some of the tasks that cannot be solved effectively by conventional simulation have become tractable by extending the simulation to operate in a symbolic domain. Symbolic simulation involves introducing an expanded set of signal values and redefining the basic simulation functions to operate over this expanded set. This enables the simulator evaluate a range of operating conditions in a single run. By linearizing the circuits with lumped parameters at particular operating points and attempting only frequency domain analysis, the program can represent signal values as rational functions in the  $s$  (continuous time) or  $z$  (discrete time) domain and are generated as sums of the products of symbols which specify the parameters of circuits elements [2 - 4]. Symbolic formulation grows exponentially with circuit size and it limits the maximum analyzable circuit size and also makes more difficult, formula interpretation and its use in design automation application [5 - 10]. This is usually improved by using semisymbolic formulation, which is symbolic formulation with numerical equivalent of symbolic coefficient. Other methods of simplification include simplification before generation (SBG), simplification during generation SDG, and simplification after generation (SAG) [11 - 17].

Symbolic response formulation of electrical circuit may be classified broadly as modified nodal analysis (MNA) [18], sparse tableau formulation and state variable formulation. The state variable method was developed before the modified nodal analysis, it involves intensive mathematical process and has major limitation in the formulation of circuit equations. Some of the limitations arise because the state variables are capacitor voltages and inductor currents [19]. The tableau formulation

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has a problem that the resulting matrices are always quite large and the sparse matrix solver is needed. Unfortunately, the structure of the matrix is such that coding these routine are complicated. MNA despite the fact that its formulated network equation is smaller than tableau method, it still has a problem of formulating matrices that are larger than that which would have been obtained by pure nodal formulation [20]. In this paper a new nodal analytical method which is structured for easy simulation is introduced.

## 2 TRANSIENT NODAL EQUATIONS

In this paper a nodal analytical method is introduced which may be used on linear or linearized RLC circuit and can be computer applicable and user friendly. The simplicity of the new transient nodal formulation lies in the fact that minimal node index is enough to formulate transient equation (1). Also standard method of building steady state nodal admittance bus is just needed to build the  $s$  - domain admittance bus while the branch source current are modified to its branch effective transient source current which comprises of the vector sum of the actual source current, its  $s$  - domain equivalent and the corresponding  $s$  - domain branch storage element induced current source due to transient inception (2). When these are done a complete transient nodal circuit equation can easily be formulated. In this paper, this new method is called  $s$  - domain nodal method of branch effective current source. Simplicity, compactness and economical is the advantage of the newly formulated mesh equation.

$$Y(s)V(s) = J_e(s) \quad (1)$$

where

$$J_e(s) = J(s) + J_l(s) \quad (2)$$

where also  $Y(s)$  is the Auxiliary admittance bus,  $s$  - domain equivalent of nodal admittance,  $J_e(s)$  is the transient nodal

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equivalent current source,  $J(s)$  is the actual steady state nodal current source vector transformed to  $s$  - domain, and  $E_i(s)$  is the transient branch sum dc induced source current mesh vector at the instant of transient inception due to the constitutive sum effect of branch storage elements on dc current flowing in these respective branches around various meshes at that instant of transient..

## 2.1 Nodal Circuit Equation Formulation

In this analysis branch companion model for R L C is derived using laplace transform and then a generalized  $s$ -domain nodal equation is configured which is essentially modeled with a modified source current, modification and derivation are follow

consider a three node circuit in fig 1, the generalization of nodal analysis of  $n$  th node may be demonstrated by forming equations of Kirchhoff's current law at the three various nodes, thus

For Node 2

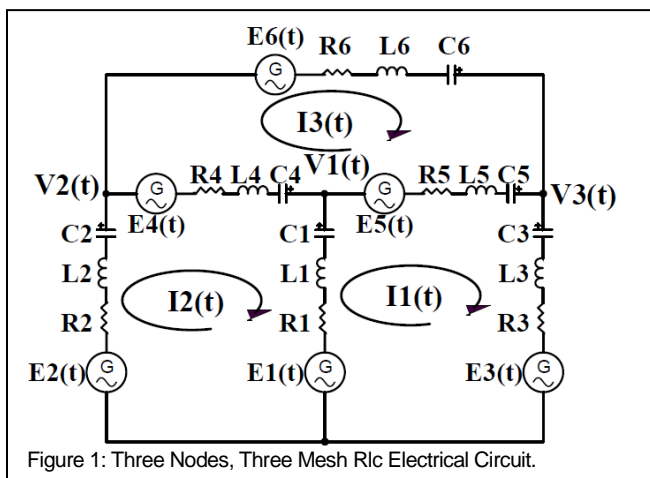


Figure 1: Three Nodes, Three Mesh Rlc Electrical Circuit.

$$i_1 + i_4 - i_5 = 0 \quad (3)$$

For  $i_4$

The constitutive effect of the branch elements is as follows

$$V_1(t) - E_4(t) - V_2(t) = R_4(t) + L_4 \frac{d}{dt} i_4(t) + \frac{1}{C_4} \int i_4(t) \quad (4)$$

taking the laplace transform of equation (4)

$$V_1(s) - E_4(s) - V_2(s) = [R_4 + sL_4 + \frac{1}{sC_4}] i_4(s) - L_4 i_4(0) + \frac{V_{C4}(0)}{s} \quad (5)$$

but

$$V_{C4}(0) = \frac{i_4(0)}{s_1 C_4} \quad (6)$$

substituting equation (6) in (5) and simplifying to get

$$i_4(s) = [V_1(s) - E_4(s) - V_2(s)] Y_4(s) - E_{(l)4}(s) Y_4(s) \quad (7)$$

$$i_4(s) = [V_1(s) - V_2(s)] Y_4(s) - E_{(e)4}(s) Y_4(s) \quad (8)$$

$$i_4(s) = [V_1(s) - V_2(s)] Y_4(s) - J_{(e)4}(s) \quad (9)$$

where

$$E_{(l)4}(s) = i_4(0) Z_{(C)4}(s)$$

$$Z_{(C)4}(s) = [L_4 - \frac{1}{ss_1 C_4}]$$

$$E_{(e)4}(s) = E_4(s) + E_{(l)4}(s)$$

$$J_{(e)4}(s) = E_{(e)4}(s) Y_4(s) \quad (10)$$

similarly  $i_1$

$$i_1(s) = V_1(s) Y_1(s) - J_{(e)1}(s) \quad (11)$$

similarly  $i_5$

$$i_5(s) = [V_1(s) - V_3(s)] Y_5(s) - J_{(e)5}(s) \quad (12)$$

The sum of the current flowing onto node 1 may be obtained by adding equation (9), (11) and (12) to get

$$V_1(s) [Y_1(s) + Y_4(s) + Y_5(s)] - V_2(s) Y_4(s) - V_3(s) Y_5(s) = J_{(e)1}(s) + J_{(e)4}(s) - J_{(e)5}(s) \quad (13)$$

For Node 2

Similarly, the Kirchhoff's current equation for node 2 may be written as follows

$$V_2(s) [Y_2(s) + Y_4(s) + Y_6(s)] - V_1(s) Y_4(s) - V_3(s) Y_6(s) = J_{(e)2}(s) - J_{(e)4}(s) - J_{(e)6}(s) \quad (14)$$

For Node 3

Similarly, the Kirchhoff's current equation for node 3 may be written as follows

$$V_3(s) [Y_3(s) + Y_5(s) + Y_6(s)] - V_1(s) Y_5(s) - V_2(s) Y_6(s) = J_{(e)3}(s) + J_{(e)5}(s) - J_{(e)6}(s) \quad (15)$$

equations (13), (14) and (15) are combined to get the nodal equation for fig 1 circuit as follows

$$\begin{pmatrix} Y_{11}(s) & Y_{12}(s) & Y_{13}(s) \\ Y_{21}(s) & Y_{22}(s) & Y_{23}(s) \\ Y_{31}(s) & Y_{32}(s) & Y_{33}(s) \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{pmatrix} = \begin{pmatrix} J_{(eN)1}(s) \\ J_{(eN)2}(s) \\ J_{(eN)3}(s) \end{pmatrix} \quad (16)$$

$$Y_{11}(s) = Y_1(s) + Y_4(s) + Y_5(s)$$

$$Y_{22}(s) = Y_2(s) + Y_4(s) + Y_6(s)$$

$$Y_{33}(s) = Y_3(s) + Y_5(s) + Y_6(s)$$

$$Y_{12}(s) = Y_{21}(s) = -Y_4(s)$$

$$Y_{13}(s) = Y_{31}(s) = -Y_5(s)$$

$$Y_{23}(s) = Y_{32}(s) = -Y_6(s)$$

$$\begin{aligned} J_{(eN)1} &= J_{(e)1}(s) + J_{(e)4}(s) - J_{(e)5}(s) \\ J_{(eN)2} &= J_{(e)2}(s) + J_{(e)4}(s) - J_{(e)6}(s) \\ J_{(eN)3} &= J_{(e)3}(s) + J_{(e)5}(s) - J_{(e)6}(s) \end{aligned} \quad (18)$$

## 2.2 GENERALIZED MATRIX FOR TRANSIENT N – TH NODAL EQUATION

Equation (16) may be used to generalize the nodal solutions n – th node in laplace frequency domain when all the branch effective source currents have been formulated as follows.

$$\begin{pmatrix} Y_{11}(s) & Y_{12}(s) & \cdots & Y_{1n}(s) \\ Y_{21}(s) & Y_{22}(s) & \cdots & Y_{2n}(s) \\ \vdots & \vdots & & \vdots \\ Y_{n1}(s) & Y_{n2}(s) & \cdots & Y_{nn}(s) \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_2(s) \\ \vdots \\ V_n(s) \end{pmatrix} = \begin{pmatrix} J_{(eN)1}(s) \\ J_{(eN)2}(s) \\ \vdots \\ J_{(eN)n}(s) \end{pmatrix} \quad (19)$$

$$J_{(eN)n}(s) = \sum_{k=1}^K J_{(e)k}(s) \quad (18)$$

$$J_{(e)k}(s) = \sum_{k=1}^K [E_k(s) + I_k(0)Z_{(C)k}(s)]Y_k(s) \quad (19)$$

$$J_{(e)k}(s) = \sum_{k=1}^K [E_k(s) + E_{(l)k}(s)]Y_k(s) \quad (20)$$

$$Z_{(C)k}(s) = [L_k - \frac{1}{ss_1C_k}], \quad s_1 = j\omega \quad (21)$$

$$E_{(e)k}(s) = E_k(s) + E_{(l)k}(s) \quad (22)$$

where

$$E_{(l)k}(s) = I_k(0)Z_{(C)k}(s) \quad (23)$$

$$J_{(e)k}(s) = E_{(e)k}(s)Y_k(s) \quad (24)$$

$$Y_k(s) = \frac{1}{R_k + sL_k + \frac{1}{sC_k}} \quad (25)$$

## 2.3 Generalized Compact Form For Transient N – Th Nodal Equation

The generalized compact form of the equation (19) is thus as follows

$$V(s)Y(s) = J_e(s) \quad (26)$$

where

$$Y(s) = \begin{pmatrix} Y_{11}(s) & Y_{12}(s) & \cdots & Y_{1n}(s) \\ Y_{21}(s) & Y_{22}(s) & \cdots & Y_{2n}(s) \\ \vdots & \vdots & & \vdots \\ Y_{n1}(s) & Y_{n2}(s) & \cdots & Y_{nn}(s) \end{pmatrix} \quad (27)$$

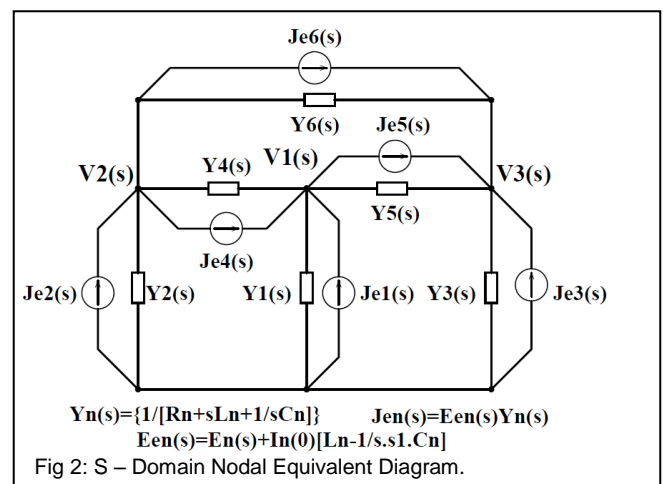
$Y(s)$  is laplace frequency domain admittance bus, the admittance could be built from fig 2 using any standard

method of building an admittance bus so far the branch admittances  $Y_k(s)$  of the circuit are evaluated as in equation (25). In this paper it is called the s – domain auxiliary impedance bus.

also

$$V(s) = \begin{pmatrix} V_1(s) \\ V_2(s) \\ \vdots \\ V_m(s) \end{pmatrix}, \quad J_e(s) = \begin{pmatrix} J_{e1}(s) \\ J_{e2}(s) \\ \vdots \\ J_{em}(s) \end{pmatrix} \quad (28)$$

$V(s)$  and  $J_e(s)$  are the vector of nodal transient voltage and transient nodal effective current source respectively, all in frequency domain.



## Definitions

$k=1, 2, \dots (K - \text{th})$  branch that are incident on  $(n - \text{th})$  node  $n=1, 2, \dots (N - \text{th})$ .

$I_k(0) \rightarrow$  is the dc current flowing in the  $(k - \text{th})$  branch at the instant of transient.

$E_k(s) \rightarrow$  is the Laplace transform of the source voltage in the  $(k - \text{th})$  branch.

$Z_{(C)k}(s) \rightarrow$  is the net transfer impedance for the transient dc quantities in the frequency domain due to the energy storing components in the  $(k - \text{th})$  branch  $Z_{(C)k}(s)$  transforms the dc current  $I_k(0)$  flowing in  $(k - \text{th})$  branch to transient dc source voltage  $E_{(l)k}(s)$  in that branch.

$E_{(l)k}(s) \rightarrow$  is the transient dc induced source voltage in the  $(k - \text{th})$  branch at the instant of transient inception due to the constitutive effect of the branch transient impedance operator  $Z_{(C)k}(s)$  on the dc current  $I_k(0)$  flowing in that branch at that instant.

$E_{(e)k}(s) \rightarrow$  is the effective source voltage in the  $(k - \text{th})$  branch due to the net effect of source voltage and the induced dc source voltage during transient in frequency domain.

$J_{(e)k}(s) \rightarrow$  is the effective source current in the  $(k - \text{th})$  branch source

current during transient in frequency domain.

$J_{(e)n}(s) \rightarrow$  is the sum of the effective source in all the (k - th) branches incident on due to the net effect of source current and the induced dc s the (n - th) node.

### 3 ANALYSIS PROCEDURES

1. Calculate the steady state branch current.
2. Transform all the branch admittances to their s - domain equivalents.
3. transform all the branch voltage sources to their s - method (use laplace transformation).
4. Calculate the branch dc driving point impedance  $Z_{(e)k}(s)$ , equation (10b) and use equation (10c) to calculate the branch dc induced branch source voltage.
5. Use equation (10c) to calculate equivalent transient current source.
6. Draw the s - domain equivalent of the RLC circuit by replacing the steady state circuit admittances with their s domain equivalents as they are calculated in step two. Also replace the branch steady state source current with their branch transient equivalent source current as calculated in step five.
7. From the s - domain transient equivalent circuit diagram, use any of the steady state method of formulating nodal equation to formulate s - domain transient nodal equation. It is worthy to note that formulated s - domain equivalent circuit is structurally a mere corollary of steady state circuit diagram. This is also true between the formulated transient nodal equation and the steady state mesh equation.
8. Form equation (16) and solve for  $V(s)$  using Cramer's rule.
9. Transform  $V(s)$  to time domain equivalent using laplace inverse transform. Eg. in Matlab,

$$V(t) = \text{ilap}(V(s)) \quad (23)$$

from this nodal voltages could easily be obtained at any instant.

### 4 TEST CIRCUIT

An earth faulted 100 kV - double end fed 100 km single transmission line was used for verification of the formulated s - domain transient nodal equation. In this analysis fault position is assumed to be 60%.

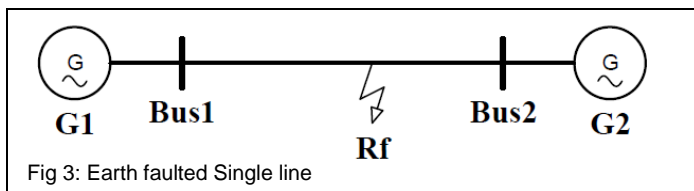


Fig 3: Earth faulted Single line

#### Test Circuit Parameters

Generator 1

$$E1(t) = 10 \times 10^4 \sin(\omega t), \quad Z_{G1} = (6 + j40) \Omega, \quad S = 1 \text{ MVA}$$

Generator 2

$$E2(t) = 0.8 |E1| \sin(\omega t + 45^\circ), \quad Z_{G2} = (4 + j36) \Omega, \quad S = 1 \text{ MVA}$$

Line Parameters

$$R_s = 0.075 \Omega/\text{km}, \quad L_s = 0.04875 \text{ H}/\text{km}$$

$$G_s = 3.75 \times 10^{-8} \text{ mho}/\text{km}, \quad C_s = 8.0 \times 10^{-9} \text{ F}/\text{km}$$

Line length = 100 km, Fault position = 60%

### 4.1 Modeling

A single pi section was adopted as a model for the test circuit. Normally the model is characterized with constant parameter, shunt capacitance of transmission line is included in the analysis while shunt conductance is neglected. The equivalent circuit of the test circuit is below fig 4.

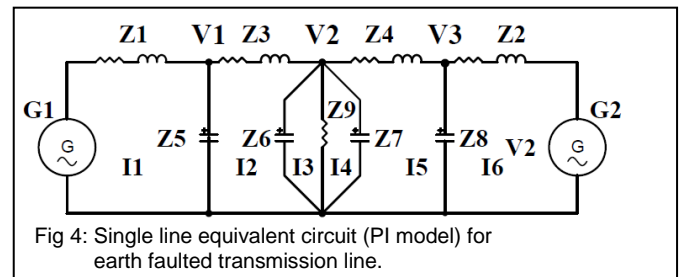


Fig 4: Single line equivalent circuit (PI model) for earth faulted transmission line.

## 5 TRANSIENT SIMULATION

### 5.1 SYMBOLIC SIMULATION WITH FORMULATED EQUATION

In this paper the transient nodal voltages were simulated by using the described formulation (s - domain nodal equation by method of branch effective transient current source). Analysis procedures of section 3 were used to calculate the s - domain rational functions of the nodal voltages (19). The obtained s - domain rational functions were transformed to close form continuous time functions using laplace inverse transformation. Discretizations of the close form continuous time functions were done to obtain to plot the nodal voltage response graphs.

### 5.2 Simpowersystem Simulation Of Test Circuit

To validate the formulated transient nodal equation, a simulation of the earth faulted double end fed, single transmission line were also performed using Matlab simpowersystem software to obtain the circuit transient nodal voltage responses. Results were compared with the responses obtained from the symbolic simulations using the formulated transient nodal equation.

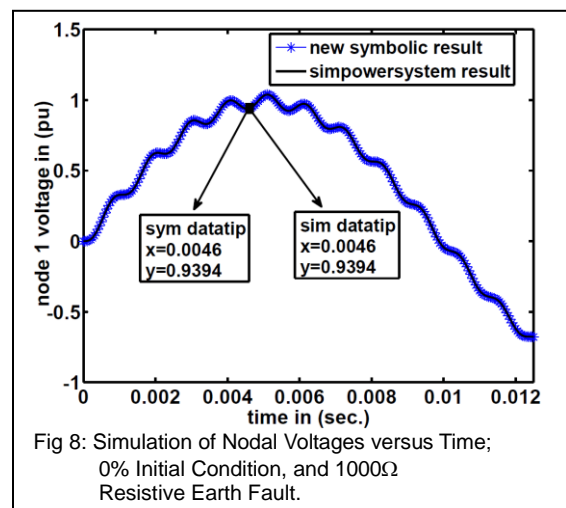
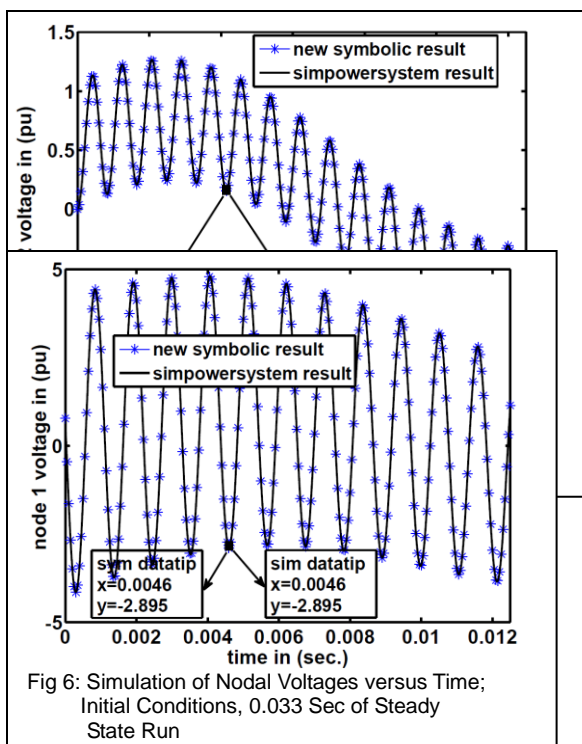
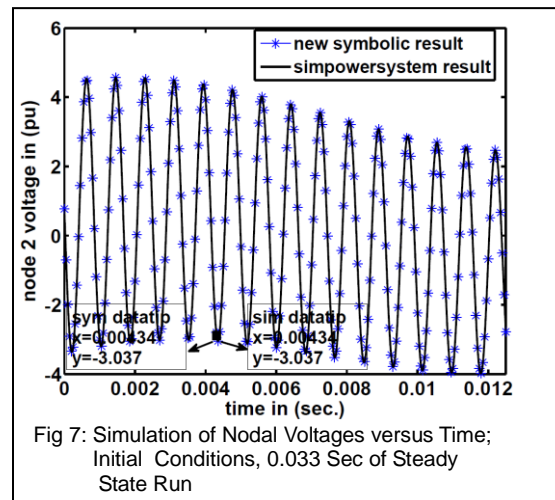
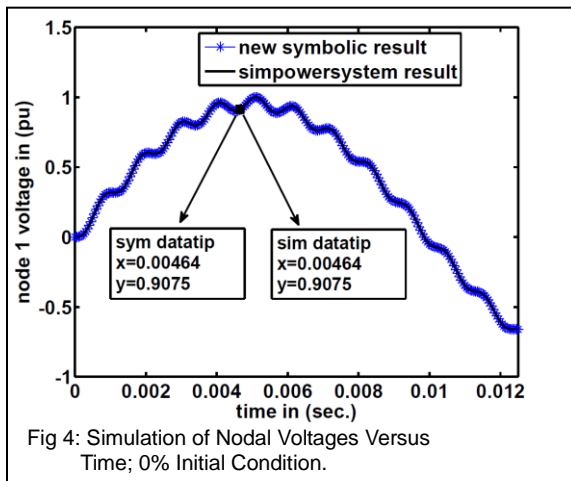
## 6 RESULTS

Nodal voltage response were simulated using the formulated nodal equation and also using simpowersystem package, all simulation were done using Matlab 7.40 mathematical tool. Simulated responses by these methods for the earth faulted

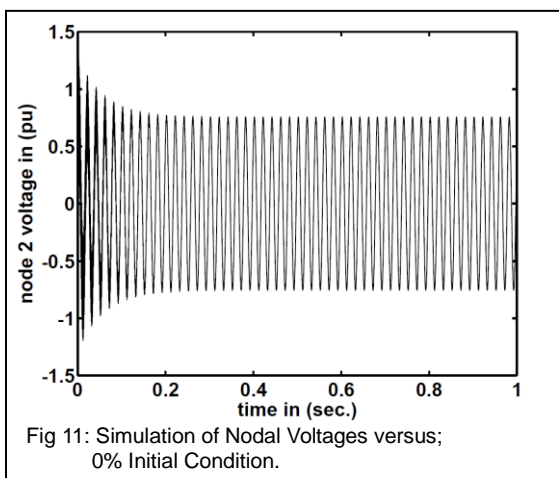
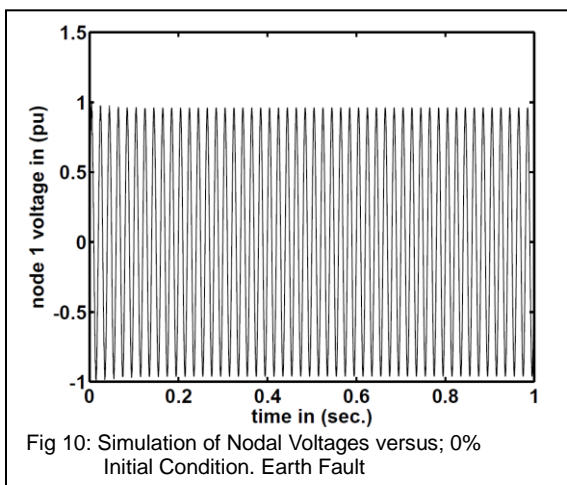
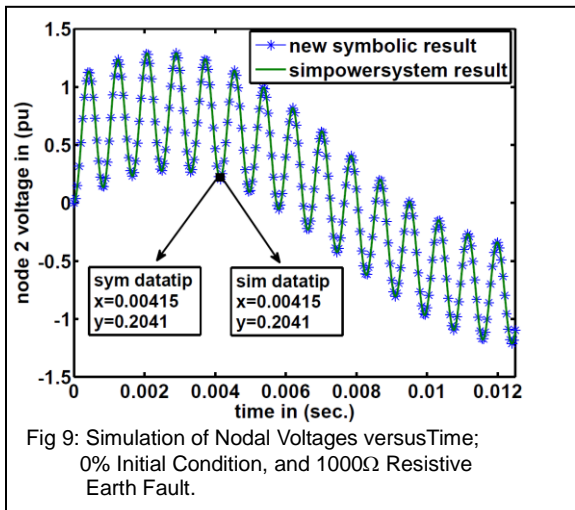
double end fed single line transmission were obtained and shown in fig 5 through fig 12. Possible data taking point of node 1 and node 3 were taken for various simulating conditions. Simulating conditions included; zero initial condition, non - zero initial condition, high resistive ( $1000\Omega$ ) fault but at zero initial condition, and 1 sec. simulation. All simulations were done, except otherwise stated on 100km line at 60% fault position and  $5\Omega$  earth resistive fault. Sampling interval for the formulated equation simulation is  $50\mu s$  while that of the simpowersystem simulation is at  $5\mu s$ . The overall result showed almost 100% conformity between new nodal symbolic formulation and the simpowersystem simulation.

## Test Circuit Simulated Nodal Voltage Response Graphs:

100 km Line, Fault Position=60% and  $5\Omega$  Resistive Earth Fault.







## 7 CONCLUSIONS

Simulation software has been formulated for transient simulation of RLC circuits initiated from steady state. The simulation software is especially useful for power circuits that are modeled with Pi - sections parameter. The result of the simulation of this new symbolic nodal software showed promising conformity with the existing simpowersystem package and has the advantage of being able to simulate imaginary initial conditions. Structurally the formulated nodal equation can easily be modified to simulate subtransient response of RLC circuits.

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